

Education
MATTERS GROUP

**Education
Matters
Tips**

MATHEMATICSTIPS & HINTSSHEETS - NUMBER & GEOMETRY

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MATHEMATICS TIPS& HINTS SHEETS

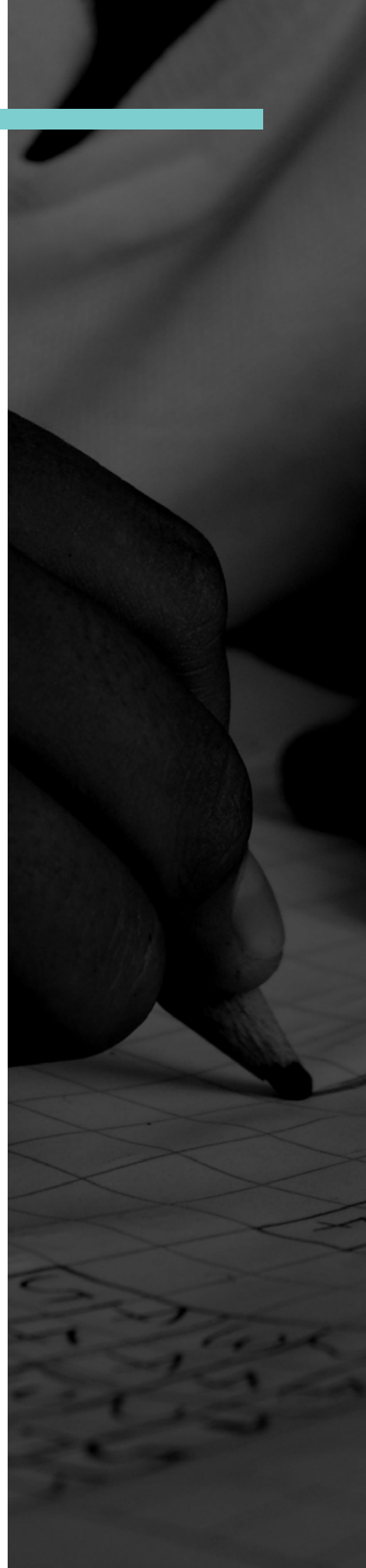
We hope this will give you some valuable hints and tips to help in teaching Mathematics concepts in a positive way.

Whilst these are some guidelines, it is important that you follow what has been set out by the school.

The tips have these topics covered-

- Place Value- Tools that could be used
- Order of operations- Associative, Commutative and Distributive Laws
- Number- Operations and Calculate
- Fractions, Decimals and Percentages
- Ratios
- Geometry- 2D and 3D shapes and their properties
- Calculating areas
- Types of lines
- Statistics& probability

We would suggest that especially with number, students move from using concrete items to using pictorial representations to written abstract methods to consolidate their learning and understanding of concepts taught.



Concrete Pictorial Abstract process

Using this process builds upon learning. Students do and see, then draw and see and then be able to write it using a written method.

Following the C-P-A-can the students show the relationship with the numbers system using manipulatives first, through pictures or drawings next and then through a written system?

Concrete

Students manipulate objects and concrete materials

Pictorial

Students draw pictures and diagrams to show learning

Abstract

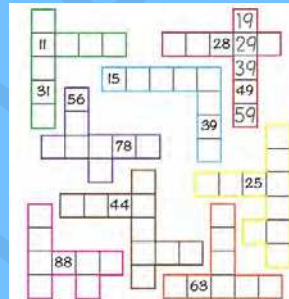
Students use numbers and mathematical symbols

Place Value

Tools that could be used- Hundreds grid, Place Value Board, Beads, Number fans, Playing cards, number jigsaw, dice, number beads, place value arrows, counters, dienes

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|-----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

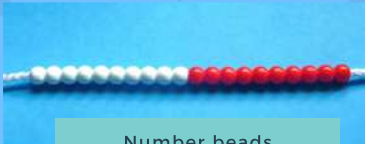
Hundreds grid



Number jigsaw



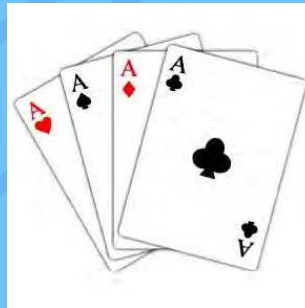
Number fans



Number beads



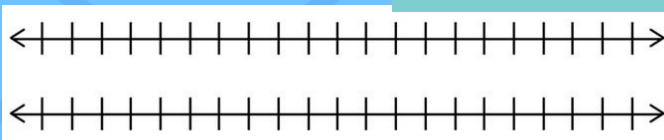
Number dice



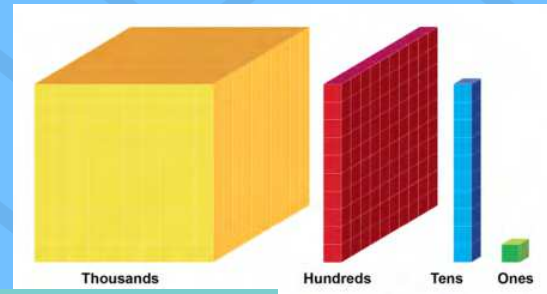
playing cards

| Hundreds | Tens | Ones |
|----------|------|------|
| | | |

place value grid



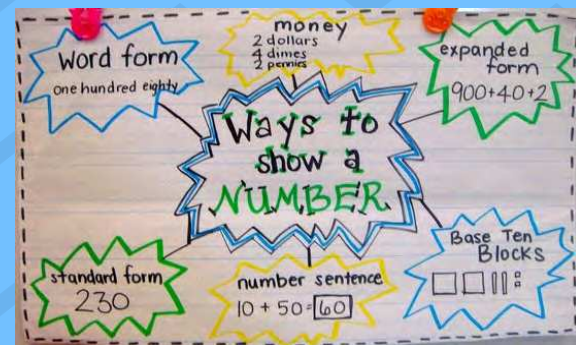
Number line



Dienes blocks

| | | |
|-----|----|---|
| 100 | 10 | 1 |
| 200 | 20 | 2 |
| 300 | 30 | 3 |
| 400 | 40 | 4 |
| 500 | 50 | 5 |
| 600 | 60 | 6 |
| 700 | 70 | 7 |
| 800 | 80 | 8 |
| 900 | 90 | 9 |
| 853 | | |

place value arrows



| BILLIONS | MILLIONS | THOUSANDS | ONES | DECIMALS |
|------------------|------------------|-------------------|----------|---------------------|
| hundred billions | hundred millions | hundred thousands | hundreds | tenths |
| ten billions | ten millions | ten thousands | tens | hundredths |
| billions | millions | thousands | ones | thousandths |
| | | | | ten thousandths |
| | | | | hundred thousandths |
| | | | | millionths |

place value grid



place value counters

Order of operations

How Do I Remember It All...? BODMAS!

B- Brackets first

O- Orders (i.e. Powers and Square Roots, etc.)

DM- Division and Multiplication(left-to-right)

AS- Addition and Subtraction(left-to-right)

Ordering Mathematical Operations

| B | O | D | M | A | S |
|-------------------|----------------------------|--------------------|----------------------------|-----------------|--------------------|
| Brackets (...) | Orders \sqrt{x} x^2 | Division \div | Multiplication \times | Addition $+$ | Subtraction $-$ |

Order of Operations

Do things in Brackets First

✓ $6 \times (5 + 3) = 6 \times 8 = 48$

✗ $6 \times (5 + 3) = \frac{30}{3} + = 33$ (wrong)

Exponents (Powers, Roots) before Multiply, Divide, Add or Subtract

✓ $5 \times 2^2 = 5 \times 4 = 20$

✗ $5 \times 2^2 = 10^2 = 100$ (wrong)

Multiply or Divide before you Add or Subtract

✓ $2 + 5 \times 3 = 2 + 15 = 17$

✗ $2 + 5 \times 3 = 7 \times 3 = 21$ (wrong)

Otherwise just go left to right

✓ $30 \div 5 \times 3 = 6 \times 3 = 18$

✗ $30 \div 5 \times 3 = 30 \div 15 = 2$ (wrong)

Concrete

Pictorial

Abstract

Following the C-P-A-can the students show the relationship with the number system using manipulatives first, through pictures or drawings next and then through a written system?

Also using the tools from the first page, think of ways you can combine the concrete examples first then get to some of the pictorial versions above

You could use - place value grid, dienes, place value counters, number lines and then progress to using pictures to explain their understanding

Distributive Law Examples

Distributive Dude!

3.MD.C.7a Multiply side lengths to find area.
3.MD.C.7c Use tiling to show concrete models of the distributive property

We use the distributive property to break apart large arrays into two smaller arrays for easier computation!

Big Array 6×4
 Little Array 1 (6×2)
 Little Array 2 (6×2)
 $(6 \times 2) + (6 \times 2) = 24$

Associative Law Examples

Definition
Changing the grouping does not change the sum or product

Addition - Example Model
 $4 + (7 + 9) = (4 + 7) + 9$

Multiplication - Example Model
 $(10 \times 2) \times 3 = 10 \times (2 \times 3)$

Non-examples
 $(15 \div 3) \div 5 \neq 15 \div (3 \div 5)$
 $(13 - 6) - 2 \neq 13 - (6 - 2)$

COMMUTATIVE PROPERTY

* You can ADD or MULTIPLY numbers in any order without changing the answer!

ADDITION EXAMPLES:
 $4 + 5 = 5 + 4 = 9$
 $6 + 3 + 4 = 3 + 4 + 6 = 13$

MULTIPLICATION EXAMPLES:
 $8 \times 3 = 3 \times 8 = 24$
 $2 \times 3 \times 4 = 4 \times 2 \times 3 = 24$

Commutative Law Examples

MULTIPLY DIVIDE

X combine equal groups (repeated addition)
÷ divide into equal groups (share equally)

Factor **factor** **product**
 $3 \times 5 = 15$

dividend **divisor** **quotient**
 $15 \div 3 = 5$

Strip Diagram
 $5 \ 5 \ 5$
 $?$

Array
 3 rows of 5

Equal Groups
 3 groups of 5

Repeated Addition
 $5 + 5 + 5 = 15$

Number Line
 (3 hops of 5)

Strip Diagram
 15
 $?$ $?$ $?$

Array
 15 split into 3 rows

Equal Groups
 share 15 equally

Repeated Subtraction
 $15 - 5 = 10$
 $10 - 5 = 5$
 $5 - 5 = 0$

Number Line
 (hop back by 5)

Multi-Digit MULTIPLICATION

Multiplication Definition:
 A mathematical operation performed on 2 or more numbers to find the **PRODUCT**. Also known as repeated addition.

Multiplication Key Words:
 product
 together
 times
 total
 multiple
 multiply
 factors
 by
 in all
 area

Grid Method

Grid Method

Since we are multiplying 2 digit by 2 digit, we need to make a 2 by 2 box.

27×65

27 is $20 + 7$
 65 is $60 + 5$

Multiply 60×20 , then 60×7 , then 20×5 , then 5×7 .

After you have found the partial products, add them all together.

1200
 420
 100
 35
 1755

Answer: 1,755

Column Method

Traditional Algorithm:

32×29

$2 \times 9 = 18$
 $9 \times 3 = 27 + 1 = 28$
 Add placeholder 0 in ones place.
 $2 \times 2 = 4$
 $2 \times 3 = 6$
 Add the Products

288
 640
 928

Open Number Line Strategies

make jumps of ten, then jumps of 1

$45 + 34$

Started at 45, jumped up 3 tens and 4 ones

Add a "friendly number" & compensate

$63 + 29$

Started at 63, jumped up 3 tens then back 1

Combine tens, then combine ones

$32 + 56$

Combined tens ($30 + 50$), then added on the combined ones ($2 + 6$)

Break up one addend

$74 + 27$

Broke second addend into $20 + 7$. Added 6 to 74 to get to 80, then added remaining 10s and 1s

Double Digit Addition WITH regrouping!

Carry the one!

Tens Ones

Start, Start, Start on the right!

43
 $+ 27$
 70

1
 4
 $+ 2$
 7

3
 7
 $+ 7$
 10

Double Digit Subtraction WITH regrouping!

Line up your Tens and ones!

Start, Start, Start on the right!

Tens Ones

56
 $- 24$
 32

5
 6
 $- 2$
 4

12
 3
 $- 2$
 8

MORE on the FLOOR, Go next door and get 10 more!

Multiplication and Division Strategies

Groups
 $6 \times 3 = 18$

Array
 6 rows of 3 = 18

Number Bond
 18

Fact Family
 $6 \times 3 = 18$
 $3 \times 6 = 18$
 $18 \div 3 = 6$
 $18 \div 6 = 3$

Repeated Addition
 $3 + 3 + 3 + 3 + 3$
 $+ 3 = 18$

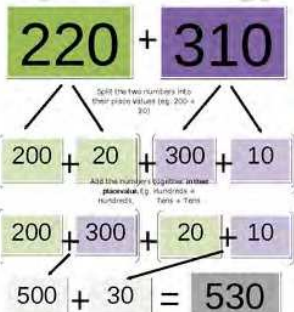
Tape Diagram
 $3 \times 6 = 18$

Skip Counting
 $3, 6, 9, 12, 15, 18$

| hundreds | tens | units |
|----------|------|-------|
| | 3 | 9 |
| | 2 | 7 |
| | 6 | |
| | 1 | |

39
 $+ 27$
 6
 1

Split Strategy



Step 1

24 into 3 does not go - carry the three over to the 4 - making it 34

Step 2

There is 10 left over, add this to the 8 to make 108

Step 3

There is 12 left over add this to the decimal 0 to make 120

$$\begin{array}{r} 12 \cdot 5 \\ 24 \overline{) 348 \cdot 120} \end{array}$$

24 x 2 = 48
24 x 4 = 96
24 x 5 = 120

Multiply the number to be divide by 2 to help you do your working out

Fraction Butterfly

multiply the digits in the blue set of wings to find the product for the blue antenna

multiply the digits in the red set of wings to find the product for the red antenna

$$\frac{16}{24} = \frac{2}{3} \times \frac{3}{8} = \frac{9}{24}$$

you can compare the fractions by comparing the products in the antennae

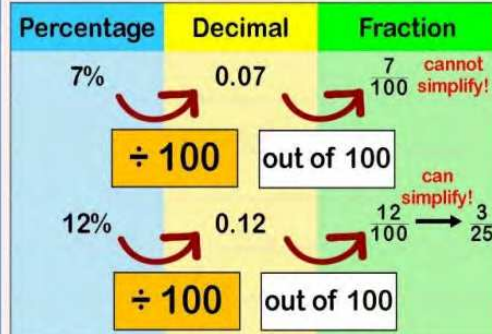
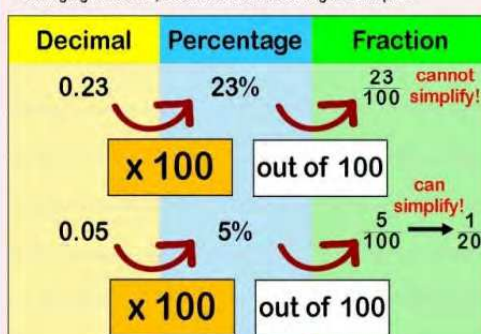
create an equivalent fraction for each original fraction using the new numerators and common denominator

add or subtract the fractions using the equivalent fractions

multiply the digits in the denominators (bottom) to find a common denominator

| Fraction | Percent | Decimal |
|----------|---------|---------|
| 1 | 100% | 1.0 |
| 1/2 | 50% | 0.5 |
| 1/3 | 33.3% | 0.33 |
| 1/4 | 25% | 0.25 |
| 1/5 | 20% | 0.2 |
| 1/6 | 16.6% | 0.166 |
| 1/8 | 12.5% | 0.125 |
| 1/10 | 10% | 0.1 |
| 1/12 | 8.3% | 0.083 |

Changing Fractions, Decimals and Percentages Examples



Division of Fractions

Step 1

$$\frac{2}{3} \div \frac{3}{7} = ?$$

Step 2

$$\frac{2}{3} \times \frac{7}{3} = \frac{2 \times 7}{3 \times 3} = \frac{14}{9}$$

Step 3

$$\frac{14}{9} \rightarrow \frac{9}{9} \times \frac{1}{5} = 1 \frac{5}{9}$$

Multiplication of Fractions

Step 1

$$\frac{2}{4} \times \frac{2}{4} = \frac{4}{16}$$

Step 2

$$\frac{2}{4} \times \frac{2}{4} = \frac{4}{16}$$

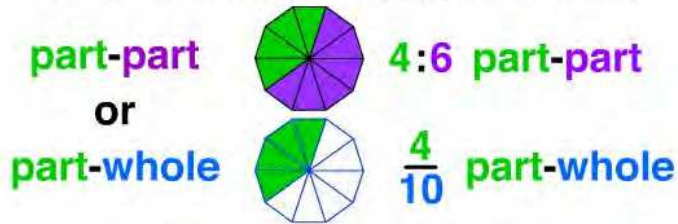
Step 3

$$\frac{4}{16} \div 4 = \frac{1}{4}$$

ratio

A ratio shows the relative sizes of two or more like values.

A ratio may compare a selected number of parts to other parts in the whole, or compare the selected number of parts to the total number of all parts in the whole.



These relative values in a ratio are often called part-part or part-whole.

A ratio can be written as:

four to six, 4 to 6, 4:6 or $\frac{4}{6}$

The numbers in a ratio are called terms.

A ratio may have more than two terms, e.g. 4:5:3:1

simplifying a ratio

Simplifying a ratio means reducing its terms to the lowest possible numbers by dividing the terms by the same number.

$800:1000 \xrightarrow{\div 10} 80:100 \xrightarrow{\div 5} 16:20 \xrightarrow{\div 2} 8:10 \xrightarrow{\div 2} 4:5$
 $800:1000 \xrightarrow{\div 10} 80:100 \xrightarrow{\div 5} 16:20 \xrightarrow{\div 2} 8:10 \xrightarrow{\div 2} 4:5$

We can use a series of small numbers, or, use the largest number possible by finding the highest common factor (HCF or GCF) to divide each term.

Determining Ratios

A Ratio is a comparison or a relationship between two items.



The Ratio of Blue Circles to Pink Circles is five blue circles compared to three pink circles.

We can write this in any three of the following ways:

Blue to Pink = 5 to 3 or 5 : 3 or $\frac{5}{3}$

Equivalent Ratios



If we double our circles we now have 10 Blue circles to 6 Pink circles. There are still 5 Blue Circles for every three Pink circles. We simply now have two of these groupings.

This means the simplest Ratio we can write for our situation is Blue to Pink = 5 : 3.

The Ratios 10 : 6 and 5 : 3 are said to be "Equivalent".

Ordering of Ratios



We can express our Ratio comparison in two ways :

Blue Circles to Pink Circles or
Pink Circles to Blue Circles

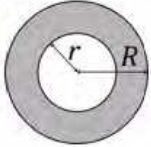
Blue to Pink = 5 to 3 or 5 : 3 or $\frac{5}{3}$

Pink to Blue = 3 to 5 or 3 : 5 or $\frac{3}{5}$

2D shapes Perimeter and Area

CIRCULAR RING

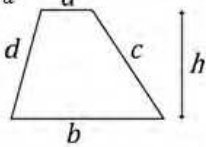
$$A = \pi(R^2 - r^2)$$



TRAPEZOID

$$P = a + b + c + d$$

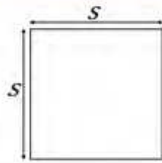
$$A = h \frac{a+b}{2}$$



SQUARE

$$P = 4s$$

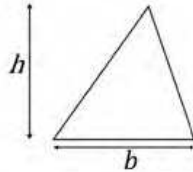
$$A = s^2$$



TRIANGLE

$$P = a + b + c$$

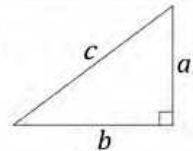
$$A = \frac{1}{2}bh$$



PYTHAGOREAN THEOREM

$$a^2 + b^2 = c^2$$

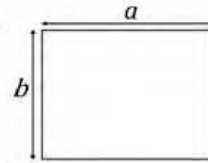
$$c = \sqrt{a^2 + b^2}$$



RECTANGLE

$$P = 2a + 2b$$

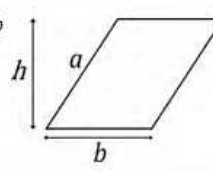
$$A = ab$$



PARALLELOGRAM

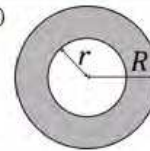
$$P = 2a + 2b$$

$$A = bh$$



CIRCULAR RING

$$A = \pi(R^2 - r^2)$$



SPHERE

$$S = 4\pi r^2$$

$$V = \frac{4\pi r^3}{3}$$

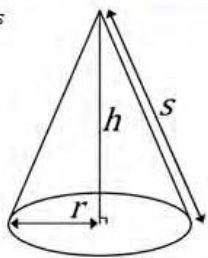


RIGHT CIRCULAR CONE

$$A = \pi r^2 + \pi rs$$

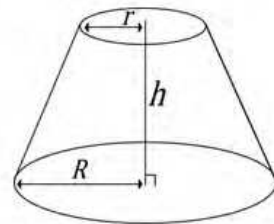
$$s = \sqrt{r^2 + h^2}$$

$$V = \frac{1}{3}\pi r^2 h$$



FRUSTUM OF A CONE

$$V = \frac{1}{3}\pi h(r^2 + rR + R^2)$$



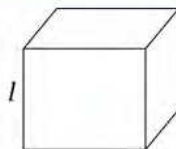
A square is a four sided regular polygon.; The rectangle is a 4 sided polygon, quadrilateral, with right angle corners.; The circle is a shape where all points along the shape are equal distance from a specific point. This point is the center of the circle and the distance to the center of the circle is the radius.; The triangle is a 3 sided polygon. Triangles can be classified by their sides: Equilateral triangles: All sides are equal in length., Isosceles triangles: Two sides are equal in length., Scalene triangles: All sides have different lengths., Triangles can also be classified by their angles: Right triangle: One angle is 90 degrees., Oblique triangle: Has no angle equal to 90 degrees., Obtuse triangle: One angle is greater than 90 degrees., Acute triangle: All angles are less than 90 degrees.; A parallelogram is a 4 sided polygon or quadrilateral with two sets of parallel sides. The opposite sides are equal in length.; A trapezoid is a 4 sided polygon or quadrilateral with one set of parallel sides.

3D shapes Area and volume

CUBE

$$A = 6l^2$$

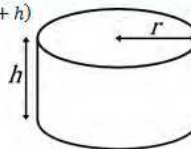
$$V = l^3$$



CYLINDER

$$A = 2\pi r(r + h)$$

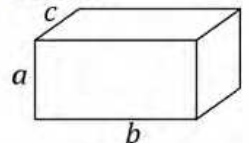
$$V = \pi r^2 h$$



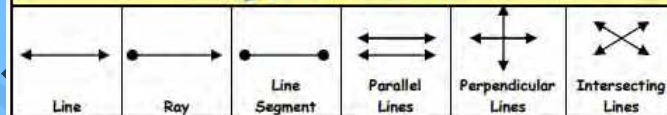
RECTANGULAR BOX

$$A = 2ab + 2ac + 2bc$$

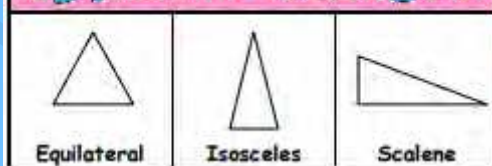
$$V = abc$$



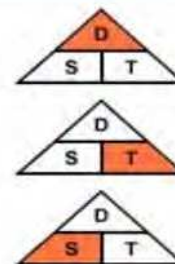
Types of Lines



Types of Triangles



Polygons



$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

The triangles will help you remember these 3 rules:

$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$\text{Time} = \text{Distance} / \text{Speed}$$

$$\text{Speed} = \text{Distance} / \text{Time}$$

Mean

Add all the numbers then divide by the amount of numbers

$$9, 3, 1, 8, 3, 6$$

$$9 + 3 + 1 + 8 + 3 + 6 = 30$$

$$30 \div 6 = 5$$

The mean is 5

Median

Order the set of numbers, the median is the middle number

$$9, 3, 1, 8, 3, 6$$

$$1, 3, 3, 6, 8, 9$$

The median is 4.5

***NB**

Mode

The most common number

$$9, 3, 1, 8, 3, 6$$

The mode is 3

Range

The difference between the highest number and lowest number

$$9, 3, 1, 8, 3, 6$$

$$9 - 1 = 8$$

The range is 8

JOINING PROBLEMS

Join (Result Unknown)
 $6 + 3 = \underline{\quad}$

Mr. Smith had 6 cookies. Suzy gave him 3 more cookies. How many cookies does Mr. Smith have now?

Join (Change Unknown)
 $4 + \underline{\quad} = 7$

Mr. Smith had 4 cookies. Suzy gave him some more. Then, Mr. Smith had 7 cookies. How many cookies did Suzy give Mr. Smith?

Join (Start Unknown)
 $\underline{\quad} + 4 = 6$

Mr. Smith had some cookies. Suzy gave him 4 more cookies. Then, he had 6 cookies. How many cookies did Mr. Smith start with?

SEPARATING PROBLEMS

Separate (Result Unknown)
 $7 - 4 = \underline{\quad}$

Mr. Smith had 7 cookies. He gave 4 of them to Suzy. How many cookies did Mr. Smith have left?

Separate (Change Unknown)
 $5 - \underline{\quad} = 1$

Mr. Smith had 5 cookies. He gave some to Suzy. Then, he had 1 cookie left. How many cookies did Mr. Smith give to Suzy?

Separate (Start Unknown)
 $\underline{\quad} - 4 = 4$

Mr. Smith had some cookies. He gave 4 to Suzy. Then, he had 4 cookies left. How many cookies did Mr. Smith have to start with?

PART - PART - WHOLE PROBLEMS

Part - Part - Whole (Whole Unknown)
 $6 + 3 = \underline{\quad}$

Mr. Smith had 6 white cookies and 3 pink cookies. How many cookies did Mr. Smith have altogether?

Part - Part - Whole (Part Unknown)
 $7 - 4 = \underline{\quad}$ or $4 + \underline{\quad} = 7$

Mr. Smith had 7 cookies. 4 were pink and the rest were white. How many white cookies did Mr. Smith have?

COMPARING PROBLEMS

Compare (Difference Unknown)
 $5 - 3 = \underline{\quad}$ or $3 + \underline{\quad} = 5$

Mr. Smith had 5 cookies. Suzy had 3 cookies. How many more cookies did Mr. Smith have than Suzy?

Compare (Quantity Unknown)
 $3 + 2 = \underline{\quad}$

Mr. Smith had 3 cookies. Suzy had 2 more cookies than Mr. Smith. How many cookies did Suzy have?

Compare (Referent Unknown)
 $8 - 5 = \underline{\quad}$

Mr. Smith had 8 cookies. He had 5 more than Suzy. How many cookies did Suzy have?

The 9 Problem Solving Strategies

1 Look for the important words in the question
Write them down.
Underline them.
Make sure I know what to do.

2 Look for a pattern
Can I see something happening over and over again?
Will this help me solve the problem?

3 Have a go
Try an answer.
Does the answer make sense?

4 Use a table or a chart
Will something like this help?

| | | | |
|--|--|--|--|
| | | | |
| | | | |
| | | | |
| | | | |

5 Use a drawing
Can I draw something about the problem?
Will this help me to find the answer?

6 Work backwards
Can I start at the end of the question to help work it out?
Will my answer work?

7 Try an easier problem
Can I change the numbers in the question to make it simpler?
Will this make finding the answer easier?

8 Make a model
Can I use paper or blocks to help me find the answer?
Can I use people to help me find the answer?

9 Think logically
Can I tell something about the answer straight away?
Can I get rid of answers that are not correct?

Problem Solving Model

Read It!

UNDERSTAND THE PROBLEM

- Read the problem 2, maybe 3 times. Highlight or underline important information.
- Talk it! – talk about the problem to understand it better.

Think It!

MAKE A PLAN

- What strategy will you use and why?
- Talk it! – discuss strategies with a partner.
- What manipulatives will you use?

Solve It!

CARRY OUT THE PLAN

- Apply your strategy.
- You may need to revise and try a different strategy....
- Show your work (thinking).
- Ask yourself...
 - Is your answer reasonable?
 - Does it make sense?

Explain It!

COMMUNICATE THE SOLUTION

- Answer the question!
- Tell, show, write, ... how the answer was reached. Consider extensions.
- First I... I noticed that... Then I... I thought... Finally...